STAT 515 Test 1

This is a 75-minute, closed-book, closed-note exam. You may use a non-programmable, non-graphing calculator. A formula sheet and normal probability table will accompany this exam. Simplify each answer according to the capabilities of your calculator.

1. $P(A) = 0.48$, $P(B) = 0.50$, and $P(A \cap B) = 0.24$

   (a) Are A and B mutually exclusive? Explain why.

   (b) Find $P(A \cup B)$.

   (c) Find $P(A|B)$.

   (d) Are A and B independent? Show your work.

2. A visitor survey of a Columbia festival indicated that 65% of attendees were from the Midlands area, and 35% were from outside the Midlands. For those from the Midlands, 30% expected to spend more than \$200 during the festival, while for those outside the Midlands, 40% expected to spend more than \$200 during the festival.

   (a) Draw a tree diagram for this problem, including all probabilities.

   (b) What is the probability a randomly-selected festival visitor plans to spend more than \$200?

   (c) Given that a visitor spends more than \$200, what is the probability that they are from the Midlands?

3. A student club with 20 members assigns two groups of students to different service activities: 10 to plant a community garden and 10 to pick up trash along a stream. Simplify your answer.

   (a) How many ways can students be assigned to the two groups?

   (b) *Graduate students only.* Suppose each of the two teams has one team leader. How many combinations of groups and group leaders are there?

   (c) Suppose another volunteer activity becomes available and students are split into three groups: 6 to plant a community garden, 8 to pick up trash along a stream, and 6 to volunteer at a local shelter. How many ways can students be assigned to the three groups?

4. A survey of coastal residents indicated that 20% comply with hurricane evacuation orders, 10% have flood insurance, and 8% comply with hurricane evacuation orders and have flood insurance.

   (a) Construct a Venn diagram for this problem and enter all probabilities.

   (b) What is the probability that a coastal resident has flood insurance and does not comply with hurricane evacuation orders?

5. Customers participating in a Pick 3 lottery may place a straight bet (pick three numbers in order) with a 0.001 chance of winning and a $500 pay off, or they may pick a 6-way box bet (pick three numbers in any order) with a 0.005 chance of winning and a $80 pay off (the actual chance is 0.006, but if a 6-way box bet lists the right order, the customer will receive the higher pay off).

   (a) What is the probability they will win nothing?

(b) Let X be the total payout. Fill out the probability table below.

x	$p(x)$

(c) What is the expected payoff?

(d) What is the variance?

6. (a) Z is a standard normal random variable. Find z_0 so that $P(Z \geq z_0) = 0.10$.

   (b) X is a normal random variable with $\mu = 80$, $\sigma^2 = 25$ and $\sigma = 5$. Find $P(75 < X < 85)$.

   (c) *Graduate students only.* X is a normal random variable with $\mu = 80$, $\sigma^2 = 25$ and $\sigma = 5$. Find x_0 so that $P(X \geq x_0) = 0.90$.

7. A student's walk between classes is normally distributed with a mean of 15 minutes and a standard deviation of 2.5 minutes.

   (a) What is the probability that the student will arrive at their next class on time (i.e., that the walk will take less than 20 minutes)?

   (b) What is the length of the commute that is exceeded only 10% of the time?

   (c) In general, do you think the time it takes to walk between classes would follow a normal distribution? Explain your answer.

8. Suppose the probability that the driver of a car has a 90% chance of wearing a safety belt. A monitor observes 15 cars drive past an observation point. Let X denote the number of drivers wearing a safety belt.

(a) What is the mean number of drivers who will be wearing a safety belt?

(b) What is the standard deviation of the number of drivers wearing a safety belt?

(c) What is the probability that all 15 drivers will be wearing a safety belt?

(d) What is $P(13 \leq X \leq 14)$?

(e) Explain why this experiment might not meet the conditions for a binomial experiment.

9. Scientists find tiny meteorites on a section of the Antarctic ice sheet at the rate of 0.5 every square kilometer.

    (a) If scientists study a 10 square kilometer area of the ice sheet in a typical field season, how many meteorites will they find on average?

    (b) What is the probability they will find exactly 5 meteorites in a 10 square kilometer area?

10. Return to the problem above.

    (a) What is the expected number of square kilometers that the scientists must search until they find their first meteorite?

    (b) What is the probability that the first meteorite will be found in the first 0.5 square kilometers searched?

Additive Rule of Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Bayes's Rule

$$P(B_i|A) = \frac{P(B_i)P(A|B_i)}{P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + \cdot + P(B_k)P(A|B_k)}$$

Discrete random variable mean and variance

$$\mu = \sum xp(x) \quad \sigma^2 = \sum (x-\mu)^2 p(x)$$

Binomial Distribution

$$\binom{n}{x} p^x (1-p)^{n-x} \qquad \mu = np \qquad \sigma^2 = np(1-p)$$

Hypergeometric Distribution

$$\frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}} \qquad \mu = n\left(\frac{r}{N}\right) \qquad \sigma^2 = n\left(\frac{r}{N}\right)\left(1-\frac{r}{N}\right)\left(\frac{N-n}{N-1}\right)$$

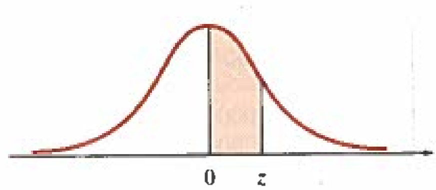
Poisson Distribution

$$\frac{\lambda^x e^{-\lambda}}{x!} \qquad \mu = \lambda \qquad \sigma^2 = \lambda$$

Exponential Distribution

$$P(X \leq x) = 1 - e^{-x\lambda} = 1 - e^{-x/\theta} \qquad \mu = \frac{1}{\lambda} = \theta \qquad \sigma^2 = \frac{1}{\lambda^2} = \theta^2$$

Table II Normal Curve Areas



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Source: Abridged from Table I of A. Hald, *Statistical Tables and Formulas* (New York: Wiley), 1952.